Adaptive Methods of Kalman Filtering for Personal Positioning Systems

E. Pulido Herrera, H. Kaufmann, *Institute for Software Technology and Interactive Systems, Vienna University of Technology*

BIOGRAPHIES

Edith Pulido Herrera received the electrical engineer degree from National University of Colombia and her Ph.D. degree in Computer Science from Universitat Jaume I (Spain). She currently holds the position of Pos Doctoral Researcher at the Interactive Media Systems Group, Institute of Software Technology and Interactive Systems at Vienna University of Technology. Her research interests include algorithms for fault detection in personal positioning systems.

Hannes Kaufmann finished his PhD thesis in 2004 on “Geometry Education with Augmented Reality”. After his postdoc research in the EU-IST project Lab@Future he got an assistant professor position at the Interactive Media Systems Group, Institute of Software Technology and Interactive Systems at Vienna University of Technology in 2005. He is now leading the VR/AR group at that institute at Vienna University of Technology. He published more than 35 scientific papers and was involved in over 10 research projects on computational geometry, geometry education, augmented reality and psychological research in AR/VR up to date.

ABSTRACT

Kalman filtering is very efficient for data fusion, in which the definition of the process and measurement noises (i.e. the matrices *Q* and *R*, respectively) greatly influences the filter performance. In recent years several studies reported that adjustments of *Q* and *R* can be helpful to reduce the errors of the estimations.

In this paper, various methods for making adjustments to the matrices *Q* and *R* are introduced for the particular case of Personal Positioning Systems (PPS). The aim is to observe the improvements achieved in an extended Kalman filter when adaptive methods are applied, in other words to observe their influence on the user's path obtained. These adjustments are considered to be needed because environmental conditions in such systems are often not fixed.

The methods to be analyzed are: (1) the weighted Kalman filter; (2) scaling matrix *Q* (3) adjustments of *Q* and *R* based on sequence-innovation; and (4) a combination of the method (2) and (3), i.e. *Q* is estimated by applying a scale factor and adjustments to *R* are realized in accordance with the method (3).

Given that the filter may diverge, we use the χ^2 test to evaluate validity of the estimations, which is based on the analysis of the innovations. Individual components of the innovation vector are evaluated in order to correct or eliminate wrong information for data fusion.

The PPS is based on the Dead Reckoning (DR) algorithm. The errors of the DR parameters are estimated with an Extended Kalman Filter (EKF), which combines the measurements of a GPS and an inertial measurement unit (IMU). The results show that each method allows us to obtain consistent Kalman filtering and they help to obtain better user’s trajectories, but additional techniques and/or technologies should be used.

INTRODUCTION

In a great number of applications, *personal positioning systems (PPS)* are very useful, e.g., in rescue work. In such applications, accurate information of the position is required from either the victim or the rescue worker (e.g. lifeguard, firefighter, etc). In PPS, sudden or unexpected changes of the behavior of humans are factors that can increase the complexity of the modeling—for example, when a person turns quickly. Hence, conducting studies and developing techniques are essential in order to have better position estimations.

Outdoor positioning information is usually provided by sensor systems, such as the global positioning system (GPS) or inertial navigation systems (INS). This information is often merged by Kalman filtering, since its efficiency for data fusion has been demonstrated. Nevertheless, the filter performance can be affected by changes in the environment or in the system dynamics [8]. Several adaptive methods have been developed in order to overcome this problem. Those methods are usually defined to adjust the filter parameters to the time varying conditions; specifically the process covariance matrix (*Q*) and the measurement noise covariance matrix (*R*). This is due to the fact that prior knowledge of the true values of *R* and *Q* is needed. However, in practice we usually have very little or no knowledge at all of the values of *R* and/or *Q*.

In this work, we focus on four methods to adapt the Kalman filter in order to obtain better information of the person’s position. The first method is named *weighted*

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Kalman filter, which was proposed by Anderson [1]. This consists of applying weighted exponential values. The second method consists of applying a *scale factor to Q* [5]. The third method is an adaptive estimation of the values of *R* and *Q* based on innovation sequence [8] [9]. The last method is a combination of the second and third approaches: *Q* is estimated by applying a scale factor and *R* is estimated by using the innovation sequence as is given in [9].

The methods mentioned above are applied to an extended Kalman filter (EKF). The EKF is used to merge the information in a personal positioning system (PPS), which is made up of a GPS receiver and an Inertial Measurement Unit (IMU). The information provided by these sensors allows us to utilize the *dead reckoning algorithm (DR)*, whose main parameters are the *stride length* of the person and the *azimuth bias*. By means of an appropriate combination, the filter estimates the errors of the DR parameters [7] to finally correct the estimations of the user's position.

To establish the validity of the estimation we utilize the χ^2 test. In this test, filter innovations are evaluated every instance; when they fail the test, one of the adaptive methods mentioned above, is applied.

POSITIONING

Localization is based on the dead reckoning algorithm (DR), whose main parameters are the stride length of the user and the azimuth. The azimuth indicates the orientation of the user's trajectory. The stride length *is the distance traveled between two consecutive foot strikes of the same foot* [14]. An important aspect to calculate the value of the stride length is to identify when a step occurs. Several methods have been proposed to solve this problem; here, we detect the peaks of the linear vertical acceleration [10] while the user walks. In this case the sensor is placed in the lower back of the person. The stride length is obtained through a neural network. We use this technique because it is a good mechanism in case the system model is not available or if modeling the system is very difficult. Details of this process can be found in [12].

Hence, the main error sources of DR are the stride length and the azimuth bias. Bias error can be due to the environment interferences, body offset, among others [10][4][7]. By applying a similar algorithm to that one given in [7] we can estimate the stride length and the bias errors. In [7] this is carried out by fusing the information provided by a GPS receiver with other sensors through an extended Kalman filter (EKF).

EKF requires the state model and the measurement model. For our PPS the state model is given by the following equations:

$$x = \begin{bmatrix} east & ve & north & vn & sl_e & b_e \end{bmatrix}^T \quad (1)$$

$$\Phi = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\beta \Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where x is the state vector whose components are: east, east velocity (ve), north, north velocity (vn), stride length error (sl_e), and bias error (b_e); Φ is the transition matrix.

The measurement model is given by:

$$z = \begin{bmatrix} \tilde{e} & \tilde{v}_e & \tilde{n} & \tilde{v}_n & \tilde{s} & \tilde{\psi} \end{bmatrix}^T \quad (3)$$

$$\begin{aligned} h &= \begin{bmatrix} east & ve & north & vn & h_5 & h_6 \end{bmatrix}^T \\ h_5 &= \left(\sqrt{ve^2 + vn^2}\right)\Delta t \\ h_6 &= \tan^{-1}(ve/vn) + B_{sensor} - b_e \end{aligned} \quad (4)$$

$$H = \frac{\partial h}{\partial x} \quad (5)$$

where z is the measurement vector whose components are: east ($\tilde{e}$), east velocity $\tilde{v}_e$, north ($\tilde{n}$), north velocity ($\tilde{v}_n$), stride length ($\tilde{s}$) and azimuth ($\tilde{\psi}$); H is the measurement matrix, which is the jacobian of *h*. The first 4 components of the vector *z* are provided by a GPS receiver, $\tilde{s}$ is obtained by the neural network and $\tilde{\psi}$ is provided by the IMU.

ADAPTIVE METHODS

The following methods are evaluated and described in the next sessions:

i. Weighted Kalman filter.
ii. Scaling *Q*.
iii. Estimation of *R* and *Q* based on the innovation sequence.

Weighted Kalman Filter

This method in combination with fuzzy logic has been proposed for INS/GPS systems [13]. It was originally an approach by Anderson [1]. It addresses the case where *R* and *Q* are unknown and it assumes them to be exponential weighted values. By assuming *R* and *Q* in this way, in fact we are actually weighting the filter in general. The resulting equations for the filter, which have been adapted from [11] are then presented.

The matrices *R* and *Q* are assumed to be:

$$R_k = R_k \delta^{-2(k+1)} \quad (6)$$

$$Q_k = Q_k \delta^{-2(k+1)} \text{, with } \delta \geq 1 \quad (7)$$

From [1] the weighted error covariance matrix is defined as:

$$(8)$$

After carrying out the corresponding transformations, the resulting equations for the prediction and correction phases are:

Prediction:

$$\hat{x}_k^- = \Phi_k \hat{x}_k \quad (9)$$

$$P_k^{\delta-} = \delta^2 \Phi_k P_k \Phi_k^T + Q_k \quad (10)$$

Correction:

gain:

$$K_k = P_k^{\delta-} H_k^T \left(H_k P_k^- H_k^T + {R_k}/{\delta^2} \right)^{-1} \quad (11)$$

$$\hat{x}_k = \hat{x}_k^- + K_k \left(z_k - H_k \hat{x}_k^- \right) \quad (12)$$

$$P_k = \left(I - K_k H_k^T \right) P_k^{\delta-} \left(I - K_k H_k^T \right)^T + K_k R_k K_k^T \quad (13)$$

The aim is to give more weight to the last measurements, thereby avoiding the use of erroneous values from previous estimations [1] and facilitating the process of obtaining a consistent filter.

Scaling Q matrix

This is an approach given in [5], which is a variation of the algorithm proposed in [6]. It consists in simply applying a scale factor to the *Q* matrix. This solution is based on the definition of the weighting of the associated error covariance *P* (a prior); in fact what happens is that when applying the scale factor to *Q*, *P* is also being scaled:

$$P_k^- = \Phi_k P_k \Phi_k^T + \lambda_k Q_k \quad (14)$$

$$\lambda_k = \frac{\vartheta_k^{-T} \vartheta_k^-}{E} \quad (15)$$

where is the filter innovation, this is the difference between the current measurement and the state predicted. If the scale factor λ is greater than 1, the new measurement will have more weight on the estimation. E is an empirical value based on the expected sum of squares of the innovation [5].

Estimation of Q and R based on Innovation Sequence

Based on the innovation sequence *R* is adapted during the process by means of [8] [9]:

$$\hat{R}_k = \hat{C}_{\upsilon_k} - H_k P_k^- H_k^T \quad (16)$$

The innovation covariance $\hat{C}_{\upsilon_k}$ is obtained in an estimation window of size N [8]:

$$\hat{C}_{\vartheta_k} = \frac{1}{N} \sum_{j=k-N+1}^{k} \vartheta_j \vartheta_j^T \quad (17)$$

The matrix *Q* is adapted at each epoch as follows:

$$\hat{Q}_k = K_k \hat{C}_{\upsilon_k} K_k^T \quad (18)$$

FAULT DETECTION

The validity of the estimation is evaluated by applying the χ^2 test [2] in which the normalized square innovation (NIS) is evaluated. This is given by [2]:

$$\iota_k = \vartheta_k^T S_k^{-1} \vartheta_k \quad (19)$$

where *S* is the covariance associated to the innovation, which is given by:

$$S_k = H_k P_k H_k^T + R_k \quad (20)$$

ι_k must have a χ^2 distribution with η degrees of freedom, which are given by the dimensions of the measurement vector. In this work every component of the innovation vector is evaluated, thus the degree of freedom is one.

EXPERIMENTS AND RESULTS

The hardware used to carry out the tests consisted of a GPS receiver from Ublox, an inertial measurement unit from Xsens and a laptop. The user wore the GPS in a bag receiver and the IMU was placed in his lower back; while he was walking data were collected by the laptop. All data collected were processed offline in Matlab. A diagram of the data flow is illustrated in Fig. 1.

The user’s trajectory was approximately 1025 m in length and consisted of straight lines and 90 degrees turns. The tests were carried out in streets in a densely populated area in the city of Vienna. The reference trajectory is illustrated in Fig. 2.

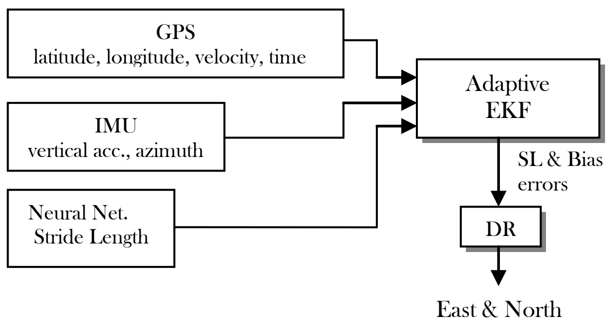

Fig. 1 Simplified block diagram of the system

Fig. 2 Reference trajectory

As the system is made up of low cost sensors, some aspects were observed in the tests. For instance, the GPS did not provide acceptable information in areas with a lot of buildings. Therefore, a lot of simulation was required to calculate initial values of *R* and *Q* in order to obtain an acceptable correction of the trajectory. Nonetheless, this does not guarantee an optimal correction, because the model used has dependence on GPS data.

In several trials the first line in the route indicates evident errors in GPS data, for instance while the user walked in a straight line the signal showed a curved route. This created a problem to determine the bias error. Likewise it causes an inappropriate behavior of the bias error in a statistical sense. To indicate this situation, we present the results for the NIS associated to the bias (denoted by ι_{bias}) and the NIS associated to the complete innovation vector (denoted by ι_v). The limit established is the value of 95% confidence level of the χ^2 distribution, which is 3.84. It can be seen that the filter had more problems when the user made a turn. This is expected because we did not include any method to correct the information in these situations and the filter does not have any self learning mechanism [3]. On the other hand, this fact allows us to evaluate the adaptive methods under these conditions (i.e. sudden change in direction of the trajectory e.g. 90 degrees turns).

Fig. 3 shows a comparison of the NIS obtained by the weighted EKF (M. 1) and the *Q* and *R* estimation based on the innovation sequence (M. 3). It is observed that in ι_{bias} many values fall outside of the allowed limits and its values were especially increased in the turns of the route. The values of ι_{bias} were considerably reduced when any of the adaptive methods was applied, as can be seen in Fig 3 (dots in green and yellow). Likewise, this provides validity to the estimations obtained by the filter.

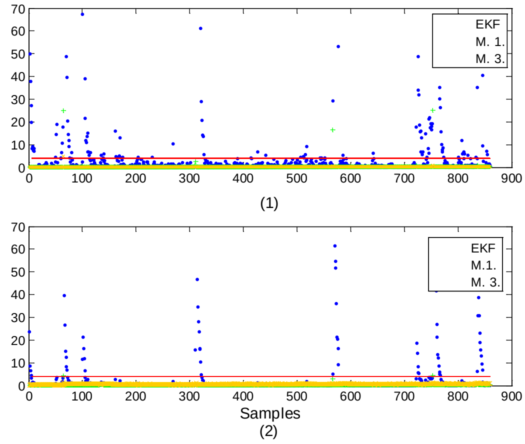

Fig. 3 Normalized innovation square for the bias error (1) and innovation vector (2). M. 1 represents the NIS obtained by the weighted EKF; M. 3 represents the NIS obtained by the adaptive method based on the innovation sequence [9]

Fig. 4 illustrates the errors over the total distance traveled in 6 simulations. The EKF, the weighted EKF and the EKF scaling Q obtained the minimal errors. However, the performance of this variable is acceptable in all methods, since in the worst case the error was 2.64%.

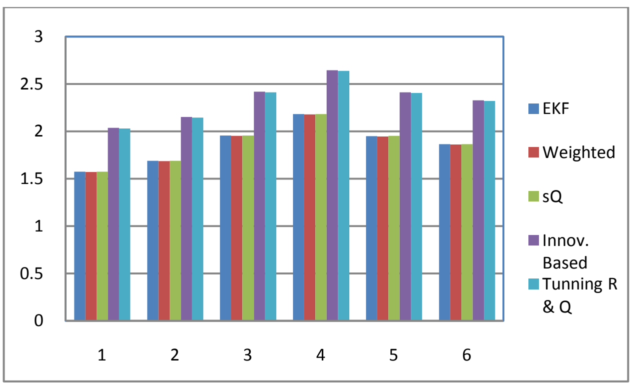

Fig. 4 Results of the error (%) over the total distance traveled in 6 simulations for each method.

The trajectories obtained by all methods are presented in Fig. 5. Several aspects can be observed in these results. First, the trajectory estimated, by correcting the errors of the DR parameters with the conventional EKF, has large errors (blue line). That implies a correction with any of the adaptive methods. According to the results obtained in the χ^2 test the variable which causes this fault was the bias error. In other words, the main problems appeared in the orientation of the trajectory. Second, as was previously mentioned in the first line of the route the GPS signal could not provide proper information to correct the trajectory. As a result it was not possible to get optimal estimations under these circumstances. Nevertheless, the methods—in which the estimation of R is based on the innovation sequence—provided a considerable improvement in this part of the path (black (M. 3) and green (M. 4) lines). Third, we can also observe the situation when the GPS can be a proper reference, for example in the third line the GPS presented roughly a straight line of the path, which can be used to estimate the bias errors.

Finally, in this test the best trajectory was obtained by the method in which R and Q are estimated based on innovation sequences (M. 3). However, a better performance of the other adaptive methods can also be achieved, but maybe depends on the GPS solution.

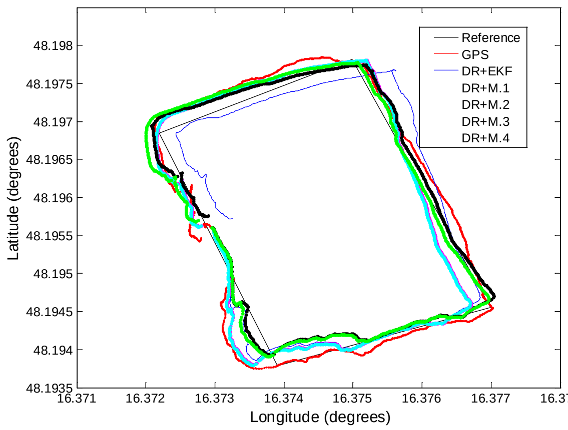

Fig. 5 Trajectories obtained by: GPS, DR in combination with EKF, M.1 (weighted EKF), M.2 (scaling Q), M.3. (adaptive estimation based on innovation sequence) and M.4 (scaling Q and R is tuned by using the technique of M.3).

Another test was carried out in order to observe the influence of the initialization in all methods. Fig. 6 shows the results obtained by the weighted EKF (M. 1) and scaling Q (M. 2); other methods were not included because they did not provide any improvement with respect to the conventional EKF. In this case the trajectories estimated by M. 1 and M. 2 are good especially when the GPS is a good reference, for example in the middle of the second line or in the third line.

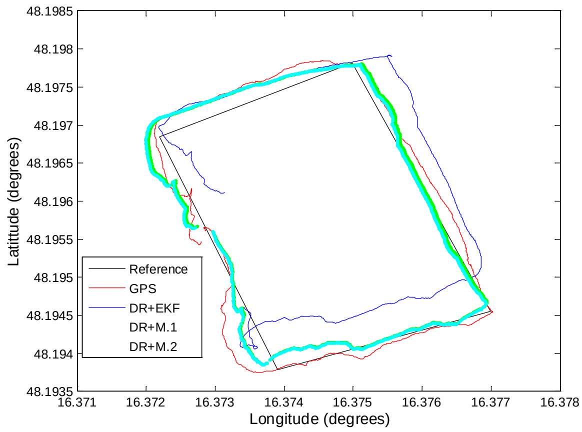

Fig. 6 Trajectories obtained by the GPS, DR in combination with: the EKF, weighted EKF (M.1) and EKF adapted by scaling Q (M.2).

Fig. 7 includes the NIS of the bias error and NIS of the innovation vector obtained in this test. In this test similar problems occurred, compared to the former test. In places where the user turned the NIS values were higher. It can also be seen that the bias error has several points outside the allowed limit. On the other hand, once the methods were applied most of the NIS passed the χ^2 test, keeping the proper performance of the filtering and at the same time a better trajectory is obtained.

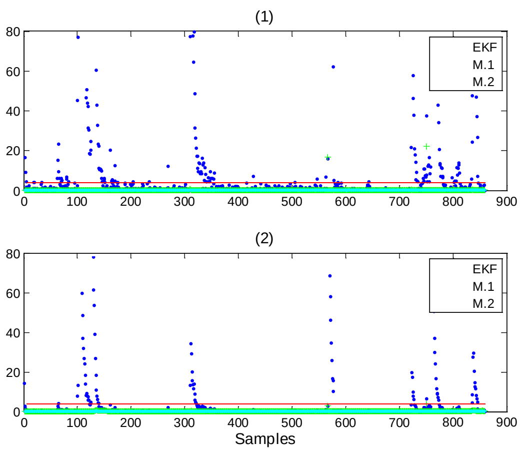

Fig. 7 (1) NIS for the error bias and (2) NIS for the innovation vector obtained by the EKF, the weighted EKF (M. 1) and scaling matrix *Q* (M. 2).

SUMMARY AND CONCLUSIONS

We mainly used three methods that showed good performance in other fields such as robotics or location vehicles. We adapted them for the particular case of personal positioning systems. These are weighted Kalman filter, scaling Q and estimation based on innovation sequences. A fourth method that makes use of principles of scaling Q and estimation based on innovation sequences was also included in the analysis. The χ^2 test was carried out in order to determine the validity of the estimations. To do this the normalized innovation square was evaluated at each instant *k*.

In our tests these methods were dependent on the initialization, in other words, it was necessary to define initial values for Q and R to achieve better estimations. The results indicated different responses of the adaptive methods according to the initialization. That is, in the first test (Fig. 5) the method that provided better estimations was M. 4 (estimation of R and Q based on the innovation sequences). In the second test with other initialization the weighted EKF and scaling Q provided better results. Each method presents some advantages, for example scaling is a very simply method, or M. 4 is a reliable method, which can provide good results as can be seen in Fig. 4 (black line). On the other hand, they also have some disadvantages, e.g. for scaling Q it could be necessary to find a self learning way to determine the proper value of E. The estimation based on the innovation sequence is a method in which we could obtain better results if some additional techniques were considered. In other words, in our tests the GPS provides unacceptable data at the beginning of the route. In such a case additional techniques based on artificial intelligence can be used, since we can take advantage of the empirical knowledge that we have. For example, it is possible to select the information provided to the Kalman filter by using fuzzy logic or neural networks.

Analyzing every variable is also an interesting aspect, because only one variable can be the source of the fault, causing erroneous data fusion. This phenomena was present in our tests, the main source of the errors in the trajectory was the bias error. We can improve robustness of the system, if we can identify the fault and correct it. In addition this information can be useful for decision-making systems.

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