Construct3D: An Augmented Reality Application for Mathematics and Geometry Education

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ABSTRACT
Construct3D is a three dimensional geometry construction tool specifically designed for mathematics and geometry education. It is based on the mobile collaborative augmented reality system "Studierstube". We describe our efforts in developing a system for the improvement of spatial abilities and maximization of transfer of learning. Means of application and integration in mathematics and geometry education at high school as well as university level are being discussed. Anecdotal evidence supports our claim that Construct3D is easy to learn, encourages experimentation with geometric constructions and improves spatial skills.

Categories and Subject Descriptors
H.5.1 [**Information Interfaces and Presentation**]: Multimedia Information Systems - *Artificial, augmented, and virtual realities*; K.3.1 [**Computers and Education**]: Computer Uses in Education - *Collaborative learning*; I.3.5 [**Computer Graphics**]: Computational Geometry and Object Modeling - Modeling packages; J.4 [**Social and Behavioral Sciences**]: Psychology.

Keywords
Mathematics education, geometry education, spatial intelligence.

Figure 1. Students working with Construct3D in our lab. In this example they inscribe a sphere in a cone. The image is generated as live video capture with computer overlays.

1. MOTIVATION
Spatial abilities present an important component of human intelligence. The term spatial abilities covers five components, spatial perception, spatial visualization, mental rotations, spatial relations and spatial orientation [3]. Generally, the main goal of geometry education is to improve these spatial skills. In a long term study by Gittler and Glück [2], the positive effects of geometry education on the improvement of spatial intelligence have been verified. Various other studies [6, 7] conclude that spatial abilities can also be improved by virtual reality (VR) technology. However, little to no work has been done towards systematic development of VR applications for practical education purposes in this field.

To fill the gap of next-generation virtual reality interfaces for mathematics and geometry education we are developing a three dimensional geometry construction tool called Construct3D [4] that can be used in high school and university education. Our system uses Augmented Reality (AR) [1] to provide a natural setting for face-to-face collaboration of teachers and students. The main advantage of using AR is that students actually see three dimensional objects which they until now had to calculate and construct with traditional (mostly pen and paper) methods. We speculate that by working directly in 3D space, complex spatial problems and spatial relationships can be comprehended better and faster than with traditional methods.

It is important to note that while geometry education software shares many aspects with conventional 3D computer-aided design (CAD) software at a first glance, its aims and goals are fundamentally different. Geometry education software is not intended for generating polished results, but puts an emphasis on the construction process itself. While relatively simple geometric primitives and operations will suffice for the intended audience of age 10 to 20, the user interface must be both intuitive and instructive in terms of the provided visualizations and tools. Commercial CAD software offers an overwhelming variety of complex features and often has a step learning curve. In contrast, geometry educators are interested in simple construction tools that expose the underlying process in a comprehensive way.

Our video demonstration shows the prototype of such an AR based geometry education tool. We present the interaction and menu system followed by an introduction of how to work with Construct3D in a one user as well as collaborative setup. A brief overview of the hardware in our stationary lab setup which provides a testbed for future evaluations is given.

2. APPLICATION DESIGN

Construct3D is based on the *Studierstube* system recently described by Schmalstieg et al. [8]. *Studierstube* uses augmented reality to allow multiple users to share a virtual space. We use see-through HMDs capable of overlaying computer-generated images onto the real world, thereby achieving a combination of virtual and real world, allowing natural communication among users. The latest version of *Studierstube* allows the mix and match of heterogeneous output devices such as personal HMD, virtual workbench, conventional monitors, and input through a variety of tracking devices. All these devices appear to act as interfaces to a single distributed system.

The current version of Construct3D offers a basic set of functions for the construction of primitives such as points, lines, planes, cubes, spheres, cylinders and cones. Construction functions include intersections, normal lines and planes, symmetry operations, and taking measurements. Recently Boolean operations based on the OpenCascade tool [5] have been added which (for instance) enable learning about intersection curves of 2^{nd} order surfaces. There is ongoing implementation work on general curves and surfaces.

Construct3D promotes and supports exploratory behavior through dynamic geometry, i. e., all geometric entities can be continuously modified by the user, and dependent entities retain their geometric relationships. For example, moving a point lying on a sphere results in the change of the sphere's radius.

All construction steps are carried out via direct manipulation in 3D using a stylus tracked with six degrees of freedom. AR affords that users see their own body and hand as well as the effects of their actions while working, so the construction process physically involves the students and resembles handcraft more than traditional computer operation. We believe that this is a key factor in the potential success of using AR for teaching geometry.

Necessary system operations such as selection of primitive type, load, delete, undo etc. are mapped to a hand-held tracked panel, the personal interaction panel (PIP) [9]. The PIP allows the straightforward integration of conventional 2D interface elements like buttons, sliders, dials etc. as well as novel 3D interaction widgets. The haptic feedback from the physical props guides the user when interacting with the PIP, while the overlaid graphics allow the props to be used as multi-functional tools.

3. EVALUATIONS

The key hypothesis - that actually seeing things in 3D and interacting with them can enhance a student's understanding of three-dimensional geometry - were supported by the anecdotal evidence we have gathered from trial runs with real students. In our first evaluation [4] with 14 students we got very positive and encouraging results and some problems were pointed out.

At this stage Construct3D is not used by students on a regular basis in mathematics and geometry education but we plan to do extensive evaluations in current and upcoming research projects where students will actually learn by using our application. While developing Construct3D we are regularly visited by teachers, students, colleagues and friends who evaluate the system and give feedback on its quality. This helps to constantly improve the application and adopt it to the students' needs.

4. FUTURE WORK

Much work remains to be done. In particular, a comprehensive evaluation of the practical value of an education tool such as ours will require the development of substantial educational content that is put to real use in classroom. We are currently at the stage where we have working tools available, but now need to apply them to real educational work. For the beginning we plan to create tutorials for vector algebra, conic sections and Boolean operations. We believe that despite the exiting possibilities of the new media, educational content creation for an interactive system is at least as difficult as authoring good textbooks, and will require a substantial amount of time and work. Finally, the true value of the new tool in classroom use needs to be verified through controlled evaluations.

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