Mathematics And Geometry Education With Collaborative Augmented Reality

Hannes Kaufmann
Interactive Media Systems Group
Vienna University of Technology
Favoritenstrasse 9-11/188
A-1040 Vienna, Austria
+43 1 58801 18860
kaufmann@ims.tuwien.ac.at

Dieter Schmalstieg
Interactive Media Systems Group
Vienna University of Technology
Favoritenstrasse 9-11/188
A-1040 Vienna, Austria
+43 1 58801 18852
schmalstieg@ims.tuwien.ac.at

Abstract

Construct3D is a three dimensional geometric construction tool specifically designed for mathematics and geometry education. It is based on the mobile collaborative augmented reality system "Studierstube". We describe our efforts in developing a system for the improvement of spatial abilities and maximization of transfer of learning. In order to support various teacher-student interaction scenarios we implemented flexible methods for context and user dependent rendering of parts of the construction. Together with hybrid hardware setups they allow the use of Construct3D in today's classrooms and provide a testbed for future evaluations. Means of application and integration in mathematics and geometry education at high school as well as university level are being discussed. Anecdotal evidence supports our claim that Construct3D is easy to learn, encourages experimentation with geometric constructions and improves spatial skills.

Keywords: augmented reality, mathematics education, geometry education, spatial intelligence

1 Motivation

Spatial abilities present an important component of human intelligence. The term spatial abilities covers five components, spatial perception, spatial visualization, mental rotations, spatial relations and spatial orientation [15]. Generally, the main goal of geometry education is to improve these spatial skills. In a long term study by Gittler and Glück [9], the positive effects of geometry education on the improvement of spatial intelligence have been verified. Various other studies [21, 24] conclude that spatial abilities can also be improved by virtual reality (VR) technology. However, little to no work has been done towards systematic development of VR applications for practical education purposes in this field.

To fill the gap of next-generation virtual reality interfaces for mathematics and geometry education we are developing a three dimensional geometric construction tool called Construct3D [11] that can be used in high school and university education (Figure 1). Our system uses Augmented Reality (AR) [1] to provide a natural setting for face-to-face collaboration of teachers and students. The main advantage of using AR is that students actually see three dimensional objects which they until now had to calculate and construct with traditional (mostly pen and paper) methods. We speculate that by working directly in 3D space, complex spatial problems and spatial relationships can be comprehended better and faster than with traditional methods.

Figure 1: Students are working with Construct3D in our standard lab setup. In the left image they inscribe a sphere in a cone, the right image shows a simple example from vector algebra. Images generated as live video capture with computer overlays.

It is important to note that while geometry education software shares many aspects with conventional 3D computer-aided design (CAD) software at a first glance, its aims and goals are fundamentally different. Geometry education software is not intended for generating polished results, but puts an emphasis on the construction process itself. While relatively simple geometric primitives and operations will suffice for the intended audience of age 10 to 20, the user interface must be both intuitive and instructive in terms of the provided visualizations and tools. Commercial CAD software offers an overwhelming variety of complex features and often has a step learning curve. In contrast, geometry educators are interested in simple construction tools that expose the underlying process in a comprehensive way.

For productive use in the classroom, a number of circumstances must be accommodated: Support for a variety of social settings including students working alone and together, a teacher working with a student or teaching a whole class, a student or the whole class taking an exam, etc. Collaboration in these situations is largely determined by roles, and the teacher should be able to retain control over the activities. Moreover, it is not realistic to expect that schools can afford extensive installations of expensive equipment such as used in our lab, and therefore the software must run on a variety of immersive and non-immersive hardware platforms including heterogeneous and hybrid setups.

This paper presents our current prototype of such an AR based geometry education tool, including hard- and software, user interface design, and initial experiences.

2 Related work

Construct3D combines four research areas: geometry, pedagogy, psychology and augmented reality. We will give a short overview of closely related work in these areas.

A large body of work has been done on 3D modeling in general. However, although 3D input devices with six degrees of freedom (6DOF) have been used to enhance modelers, little modeling has been done in immersive virtual reality systems. A good overview of 3D modeling systems with 6DOF input devices can be found in the work of Mine [18]. Mine's CHIMP [19] was used to study user interaction techniques. Similar goals were driving the work on DesignSpace by Chapin [7], Bowman's Conceptual Design Space [4] and SeamlessDesign by Kiyokawa [12].

A large number of researchers have been working on virtual reality applications for pure educational use [3, 5, 6, 28 and many others]. A very good summary of existing educational applications is given by Mantovani [16]. We want to point out CyberMath [27] which is an avatar-based shared virtual environment aimed at improving mathematics education, combining the areas of virtual reality, pedagogy and mathematics. Unlike our AR approach, CyberMath is intended for remote rather than face-to-face collaboration, and currently uses desktop VR with no support of immersive displays. In contrast, dedicated educational dynamic geometry desktop applications such as Geometer's Sketchpad [10], Cindarella [23], Euklid [17] and Cabri Geometry [13] support two-dimensional geometry only.

Regarding spatial intelligence, a recent article by Durlach et al. [8] gives a very good overview of work that has already been done in the research area of enhancing spatial skills within virtual environments but mainly identifies the indispensable need for comprehensive future research in this area. Interesting work on gender differences has been done by Rizzo et al. [24] and Larson et al. [14]. Gittler and Glück [9] study how courses in descriptive geometry improve students' spatial intelligence. The main component of their results is that there is a significant positive effect of descriptive geometry instruction on performance in spatial ability tasks.

Finally, augmented reality is a rapidly evolving area of computer graphics and user interface research. A good overview is given in the survey by Azuma [1] and its recent update by Azuma et al. [2]. However, despite its obvious appeal for face-to-face collaboration, we are unaware of any AR application directly concerned with geometry education.

3 Application design

3.1 Basic construction functions

Construct3D is based on the *Studierstube* system recently described by Schmalstieg et al. [25]. *Studierstube* uses augmented reality to allow multiple users to share a virtual space. We use see-through HMDs capable of overlaying computer-generated images onto the real world, thereby achieving a combination of virtual and real world, allowing natural communication among users. The latest version of *Studierstube* allows the mix and match of heterogeneous output devices such as personal HMD, virtual workbench, conventional monitors, and input through a variety of tracking devices. All these devices appear to act as interfaces to a single distributed system.

The current version of Construct3D offers a basic set of functions for the construction of primitives such as points, lines, planes, cubes, spheres, cylinders and cones. Construction functions include intersections, normal

lines and planes, symmetry operations, and taking measurements. Boolean operations based on the OpenCascade tool [20] have been added which (for instance) enable learning about intersection curves of 2nd order surfaces. Currently new functions for constructing general and special curves (e.g. conic sections, B-Splines) and surfaces (e.g. quadrics, B-Spline surfaces) are being implemented.

Construct3D promotes and supports exploratory behavior through dynamic geometry, i. e., all geometric entities can be continuously modified by the user, and dependent entities retain their geometric relationships. For example, moving a point lying on a sphere results in the change of the sphere's radius.

All construction steps are carried out via direct manipulation in 3D using a stylus tracked with six degrees of freedom. AR affords that users see their own body and hand as well as the effects of their actions while working, so the construction process physically involves the students and resembles handcraft more than traditional computer operation. We believe that this is a key factor in the potential success of using AR for teaching geometry.

Necessary system operations such as selection of primitive type, load, delete, undo etc. are mapped to a hand-held tracked panel, the personal interaction panel (PIP) [26]. The PIP allows the straightforward integration of conventional 2D interface elements like buttons, sliders, dials etc. as well as novel 3D interaction widgets. The haptic feedback from the physical props guides the user when interacting with the PIP, while the overlaid graphics allow the props to be used as multi-functional tools.

3.2   Layers

We are using a 3D-layer system very similar to the one used in current image editing programs. Our 3D-layers offer the possibility to arrange parts of a construction into overlapping sub-spaces that can be controlled independently. This feature is particularly powerful in conjunction with distributed multi-user operation, where every user has a personal display for which visibility of layers can be controlled independently.

The current implementation allows the arbitrary selection of visibility per user and per layer. We have implemented three basic modes:

- independent mode, i. e., every student can only see the elements constructed by himself
- collaborative mode, i. e., everything is visible to everybody
- teacher mode, i. e., a special user – the teacher – can set visibility with a user/layer matrix of controls on the PIP.

Consider a teacher working on a construction with students watching him and work on the model themselves by request. The whole construction is visible to all users. If later the teacher wants the students to practice on the same construction again, he switches to "independent" mode while the application is still running. Now each student can only see the immutable specification and the elements that he constructed himself without being influenced by the work of the teacher or fellow students. If needed, the teacher is able to switch his own construction or a reference solution on again so that some or all students can see it. The full solution to a construction task can be available from the beginning for reference purposes in a separate set of layers, and be progressively revealed by the teacher.

4   Hybrid hardware setups

To complement the diverse teacher-student interaction scenarios that are possible on the software side with practical hardware solutions for an educational environment we created various hybrid hardware setups. Realistically not all scenarios can be done in schools with equipment similar to our standard lab equipment of rather expensive tracking systems, head mounted displays and stereoscopic video projections. However, many components such as PC workstations with accelerated graphics and inexpensive projection systems are becoming feasible for classroom use. We are evaluating the following hardware setups:

4.1   The Augmented Classroom

The setup consists of two wearable augmented reality kits composed of back pack computer, stereoscopic see-through head mounted display with camera, and custom pinch gloves for two-handed input. One kit can be worn by the teacher, the second one is available for use by students. Both users can move around freely, since the kits are equipped with battery power for all devices and wireless LAN cards for communication. Furthermore, there is a small table, serving as a place for collaboration between the two users (see Figure 3, middle).

While this setup allows for first-class experiences on the students' side, the number of available AR sets significantly restricts the use in larger groups. This situation is somewhat analogous to the use of a blackboard in class: Either the teacher or a single selected student work on the blackboard, while the remainder of the class watches or works along on paper. During a lesson, students take turns at the blackboard. With the aid of an additional computer with video camera and video projection screen, we can mimic this classroom procedure by

projecting a live (monoscopic) video of the users (teacher/student) augmented with their current construction on a projection screen next to the users for the remainder of the class to watch (Figure 2).
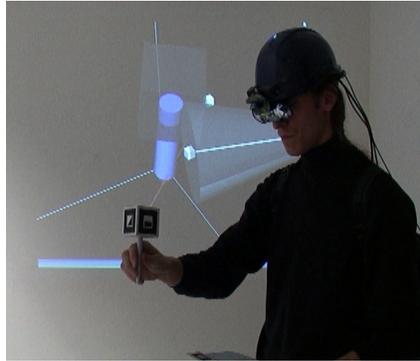

Figure 2: A teacher is working in Construct3D with the mobile AR setup while a live monoscopic video of his current construction is projected onto a projection screen behind him.

Just like in conventional classrooms, students can take turns at using the HMD and working in front of the class. To enhance the classroom situation for students not wearing an augmented reality kit, the overhead projection can also be used to view 3-dimensional content attached to markers. By moving a marker in front of the projection surface, its contents are shown on the projection (Figure 3).
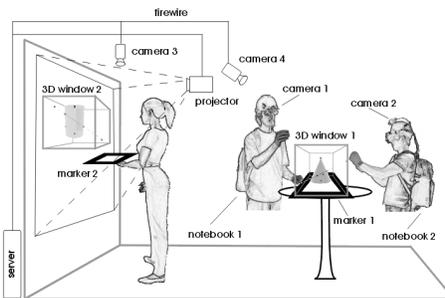

Figure 3: Left: Demonstration of the mobile AR kit at a local science fair. Middle: Schema of the Augmented Classroom setup. Two mobile users interact with a construction while a third user inspects a finished model on a projection screen. Right: Interacting with models in front of a projection screen.

It is intended to be used by high school students and teachers in an interactive, collaborative manner and to blend seamlessly into an everyday classroom situation. Tangible, tool based interaction provides a simple and intuitive user interface. Support for multiple users and spontaneous collaboration encourages team work and simplifies supervision. The system integrates mobile augmented reality, collaboration, and a tangible user interface.

The *Studierstube* platform on which the Augmented Classroom is built brings together advanced augmented reality features in a unique way. A strong distributed shared scene graph infrastructure enables collaboration between independent mobile AR kits. Dynamic loading and sharing of multi-tasked AR applications between several hosts together with support for tool based interaction allow the users to load and share constructions by handling tangible markers, to print a snapshot of their work or to save it to file. The combination of these features into a single system allows the simple development and setup of a complex application like the Augmented Classroom.

4.2 Projection screen classroom

A popular semi-immersive technique is to use just a large screen projection shared by a group of users (in our case, the class), typically showing stereoscopic images using active or passive stereo glasses. The disadvantage is that since the screen is shared between the active user (e. g., teacher, demonstrator) and the observers, head-tracking is not useful, and consequently stereoscopic images are often severely distorted if rendered for an "averaged" viewpoint. In consequence, manipulation even with tracked input devices becomes indirect (comparable to screen and mouse manipulation) as objects do not appear aligned or superimposed with the users

hands. Advantages of this approach include lower system complexity/cost, and the avoidance of cumbersome HMDs. Despite the shortcomings, projection walls are established techniques for semi-immersive group environments, and single-projector displays are affordable for classroom use.

4.3   Distributed hybrid classroom

Just like the hybrid AR classroom, this setup may use personal HMDs for realizing AR for the teacher and selected students. However, the students are all equipped with personal workstations displaying desktop VR watching the construction process on their screen. We built a desktop VR system using a FireWire camera for optical tracking and a standard consumer graphics card with shutter classes to get stereo rendering with optically tracked 6DOF input devices at a very low price (see Figure 4). The advantage of this scenario lies in the relatively low price for a personalized semi-immersive display: Students can choose individual viewpoints, maybe even manipulate local copies of the constructed object. However, a teacher can also choose a guided mode, e. g., by locking the students' views to the teacher's viewpoint.

Figure 4: A user working with our desktop VR system. A FireWire camera (out of view) is used for optical tracking of the hand held props which are equipped with markers (see yellow ellipse). The image of the camera is used as a video background. Stereoscopic images are displayed on the monitor which give the user who wears shutter glasses the impression of working in 3D space. The virtual images of pen and PIP can be seen on the monitor (red ellipse) as an overlay over the video image.

4.4   Remote collaboration

Although the advantages of co-located collaboration are lost, the same systems can be used for remote collaboration through a remotely shared 3D space. For example, a teacher can remotely advise a student at a homework problem by the same guided construction techniques as in the AR-classroom scenario, or multiple students can remotely work together. Each of the users has an individual choice of input and output facility, e. g., one user may wear a HMD, while another one uses a desktop VR setup. We are currently evaluating this possibility using our latest mobile AR hardware prototype [22] as a test platform.

5   Evaluations

The key hypothesis - that actually seeing things in 3D and interacting with them can enhance a student's understanding of three-dimensional geometry - were supported by the anecdotal evidence we have gathered from trial runs with real students. In our first evaluation [11] with 14 students we got very positive and encouraging results and some problems were pointed out. During the evaluation it was gratifying for us to see users work with Construct3D in a very constructive manner. It was obvious that they did not need a long introduction to the system but applied their experience with 2D user interfaces to the 3D interface. The students' interactions with the system in our HMD based lab setup were interesting to watch. After completing their task, some walked around the objects, viewing them from different sides or got down on their knees and looked at the scene from below. It was clear that they were proud of what they had "built". Half of the students felt that working with Construct3D for the first time is easier than a first experience with a desktop CAD package. All except one could imagine working with Construct3D without having worked with a traditional CAD package before.

Hand-eye coordination is very difficult when spotting a point accurately in 3D space without haptic feedback or constraints. All students reported problems with setting points at given coordinates. As a consequence we implemented raster and grid functions. About constructing in VR, students especially liked walking around and inside objects, the "playful" way of constructing and that spatial relationships and complex three dimensional

situations are directly visible. The clearness of Construct3D's menu system and the audio help system were mentioned positively.

Students mentioned the following possible application areas: interactive conic sections, vector analysis, enhancing spatial abilities, intersection problems, experiencing space (for very young students) and building three dimensional worlds from two dimensional views, elementary geometry, visualisation of constructions, geometry didactics - learning by doing and training of spatial abilities by viewing objects from different sides.

At this stage Construct3D is not used by students on a regular basis in mathematics and geometry education but we plan to do extensive evaluations in current and upcoming research projects where students will actually learn by using our application. While developing Construct3D we are regularly visited by teachers, students, colleagues and friends who evaluate the system and give feedback on its quality. This helps to constantly improve the application and adopt it to the students' needs.

6   Conclusions and future work

In this paper we present a fully functional educational AR application for mathematics and geometry education. We implemented flexible methods to support various teacher-student interaction scenarios. Hybrid hardware setups allow the use of Construct3D in today's classrooms and provide a testbed for these scenarios. Initial evaluations of the concept are encouraging, and our mid- to long-term plans are to integrate it in Austrian high school and higher education curricula. We have established partnerships with Austrian schools and scientific partners such as the Institute of Geometry at Vienna University of Technology and the Institute of Psychology at the University of Vienna.

Much work remains to be done. In particular, a comprehensive evaluation of the practical value of an education tool such as ours will require the development of substantial educational content that is put to real use in classroom. We are currently at the stage where we have working tools available, but now need to apply them to real educational work. For the beginning we plan to create tutorials for vector algebra, conic sections and Boolean operations. We believe that despite the exiting possibilities of the new media, educational content creation for an interactive system is at least as difficult as authoring good textbooks, and will require a substantial amount of time and work. Finally, the true value of the new tool in classroom use needs to be verified through controlled evaluations.

Acknowledgements

References

[1]   Azuma R. A Survey of Augmented Reality. *PRESENCE: Teleoperators and Virtual Environments,* Vol. 6, No. 4, pp. 355-385, 1997.

[2]   Azuma R, Baillot Y, Behringer R, Feiner S, Julier S, and MacIntyre B. Recent Advances in Augmented Reality. *IEEE Computer Graphics & Applications,* Vol. 21, No. 6, pp. 34-47, 2001.

[3]   Bell JT, and Fogler HS. The Investigation and Application of Virtual Reality as an Educational Tool. In *Proceedings of the American Society for Engineering Education 1995 Annual Conference*, Session number 2513, Anaheim, CA, 1995.

[4]   Bowman DA. Conceptual Design Space - Beyond Walk-through to Immersive Design. In Bertol, D., *Designing Digital Space*, John Wiley & Sons, New York, 1996.

[5]   Bricken M, and Byrne C. Summer Students in Virtual Reality: A Pilot Study on Educational Applications of VR Technology. In A. Wexelblat (Ed.) *Virtual Reality, Applications and Explorations*. Cambridge, MA: Academic Press Professional, 1993.

[6]   Byrne CM. *Water on Tap: The Use of Virtual Reality as an Educational Tool*. Unpublished Ph.D. thesis, University of Washington, College of Engineering, 1996.

[7]   Chapin WL, Lacey TA, and Leifer L. DesignSpace: A Manual Interaction Environment of Computer Aided Design. In *Proceedings of the ACM's SIGCHI 1994 Conference: CHI'94 Human Factors In Computing Systems*, 1994.

[8]   Durlach NI, Allen G, Darken R, Garnett RL, Loomis J, Templeman J, and von Wiegand TE. Virtual Environments and the Enhancement of Spatial Behavior: Towards a Comprehensive Research Agenda. In *PRESENCE - Teleoperators and Virtual Environments*, Vol. 9, No. 6, pp. 593-615, 2000.

[9]   Gittler G, and Glück J. Differential Transfer of Learning: Effects of Instruction in Descriptive Geometry on Spatial Test Performance. *Journal of Geometry and Graphics*, Volume 2, No. 1, 71-84, 1998.

[10] Jackiw N. *The Geometer's Sketchpad Version 3*. Key Curriculum Press, Berkeley, 1995.

[11] Kaufmann H, Schmalstieg D, and Wagner M. Construct3D: A Virtual Reality Application for Mathematics and Geometry Education. *Education and Information Technologies* 5:4, pp. 263-276, 2000.

[12] Kiyokawa K, Takemura H, and Yokoya N. SeamlessDesign for 3D Object Creation, *IEEE MultiMedia magazine*, 7 (1), pp. 22-33, 2000.

[13] Laborde JM, and Bellemain F. *Cabri-Geometry II*. Texas Instruments. Copyright Texas Instruments and Université Joseph Fourier, CNRS, 1998. URL: http://www-cabri.imag.fr/index-e.html

[14] Larson P, Rizzo AA, Buckwalter JG, van Rooyen A, Kratz K, Neumann U, Kesselman C, Thiebaux M, and van der Zaag C. Gender Issues in the Use of Virtual Environments. In *CyberPsychology and Behavior*, **2**(2), 1999.

[15] Maier PH. *Räumliches Vorstellungsvermögen*. Peter Lang GmbH, Europäische Hochschulschriften: Reihe 6, Bd. 493, Frankfurt am Main, 1994.

[16] Mantovani F. VR Learning: Potential and Challenges for the Use of 3D Environments in Education and Training. In *Towards CyberPsychology: Mind, Cognitions and Society in the Internet Age*, Giuseppe Riva & Carlo Galimberti (Eds.), Amsterdam, IOS Press, 2001.

[17] Mechling R. Euklid Dynageo, 2000. URL: http://www.dynageo.com

[18] Mine M. Working in a Virtual World: Interaction Techniques Used in the Chapel Hill Immersive Modeling Program. UNC Chapel Hill Computer Science Technical Report TR96-029, 1996.

[19] Mine M. ISAAC: A Meta-CAD System for Virtual Environments. *Computer-Aided Design*, 29(8), 1997.

[20] OpenCascade 4.0. Open-Source Toolkit for 3D modelling, 2001. URL: http://www.opencascade.com

[21] Osberg K. Spatial Cognition in the Virtual Environment, *Technical R-97-18*. Seattle: Human Interface Technology Lab, 1997.

[22] Reitmayr G, and Schmalstieg D. Mobile Collaborative Augmented Reality. In *Proceedings of the 2nd ACM/IEEE International Symposium on Augmented Reality* (ISAR'01), New York NY, 2001.

[23] Richter-Gebert J, and Kortenkamp UH. *The Interactive Geometry Software Cinderella: Version 1.2* (Interactive Geometry on Computers), 1999. URL: http://www.cinderella.de/

[24] Rizzo AA, Buckwalter JG, Neumann U, Kesselman C, Thiebaux M, Larson P, and van Rooyen A. The Virtual Reality Mental Rotation Spatial Skills Project. In *CyberPsychology and Behavior*, 1(2), pp. 113-120, 1998.

[25] Schmalstieg D, Fuhrmann A, Hesina G, Szalavari Z, Encarnação M, Gervautz M, and Purgathofer W. The Studierstube Augmented Reality Project. In *PRESENCE - Teleoperators and Virtual Environments,* MIT Press, 2002.

[26] Szalavári Z, and Gervautz M. The Personal Interaction Panel - A Two-Handed Interface for Augmented Reality, *Computer Graphics Forum*, 16, 3, pp. 335-346, 1997.

[27] Taxén G, and Naeve A. CyberMath: Exploring Open Issues in VR-Based Learning. SIGGRAPH 2001 Educators Program. In *SIGGRAPH 2001 Conference Abstracts and Applications*, 49-51, 2001.

[28] Winn W. The Impact of Three-Dimensional Immersive Virtual Environments on Modern Pedagogy. *HITL Technical Report R-97-15*. Discussion paper for NSF Workshop. Human Interface Technology Laboratory, University of Washington, 1997.

Figure 1: Students are working with Construct3D in our standard lab setup. In the left image they inscribe a sphere in a cone, the right image shows a simple example from vector algebra. Images generated as live video capture with computer overlays.
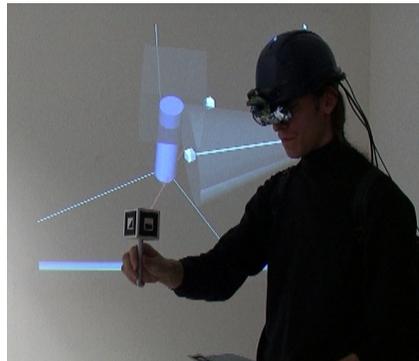
Figure 2: A teacher is working in Construct3D with the mobile AR setup while a live monoscopic video of his current construction is projected onto a projection screen behind him.
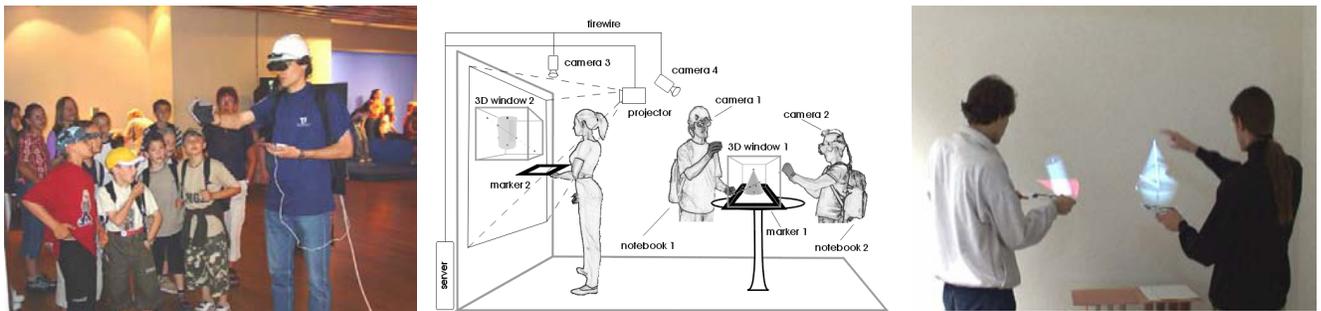

Figure 3: Left: Demonstration of the mobile AR kit at a local science fair. Middle: Schema of the Augmented Classroom setup. Two mobile users interact with a construction while a third user inspects a finished model on a projection screen. Right: Interacting with models in front of a projection screen.

Figure 4: A user working with our desktop VR system. A FireWire camera (out of view) is used for optical tracking of the hand held props which are equipped with markers (see yellow ellipse). The image of the camera is used as a video background. Stereoscopic images are displayed on the monitor which give the user who wears shutter glasses the impression of working in 3D space. The virtual images of pen and PIP can be seen on the monitor (red ellipse) as an overlay over the video image.